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TEMPUS PROJECT IB-JEP-25054-2004
Training Centre for Actuaries and Financial Analysts

ABSTRACTS

INTERNATIONAL SUMMER SCHOOL
"INSURANCE AND FINANCE: SCIENCE, PRACTICE AND EDUCATION"

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Foros (Crimea, Ukraine)

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Organizers:

Kyiv Taras Shevchenko National University (Ukraine)
Mälardalen University (Sweden)
Training Centre for Actuaries and Financial Analysts
Society of Actuaries of Ukraine
State Commission for Regulation of the Financial Services Markets of Ukraine

Organizing Commettee:

Yulia Misura (Kyiv, Ukraine)-co-chairman
Dmitrii Silvestrov (Västerås, Sweden)- co-chairman
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Evelina Silvestrova (Västerås, Sweden)
Nadija Zinchenko (Kyiv, Ukraine)
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International Summer School "**Insurance and Finance: Science, Practice and Education"** is held under the support of the EU within the framework of the EU TEMPUS PROJECT IB-JEP-25054-2004 "Training Centre for Actuaries and Financial Analysts "

Main topics:

- Actuarial Sciences
- Analytical Methods in Finance
- Educational Actuarial Programs
- Training Centre for Actuaries and Financial Analysts
- Problems of Insurance Business in Ukraine and Actuarial Support of Insurance Companies Activity.

INVITED LECTURES

OPTIMAL INVESTMENT FOR INSURER IN MODEL WITH VARIABLE PREMIUM INCOME

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We consider a risk process modelled as a combination of a variable premium income process with a compound Poisson process of claims. The ruin probability of risk process is minimized by the choice of a value of investment into the risky asset. The optimal strategy is computed using Hamilton-Jacobi-Bellman equation.

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CONTENT OF TRAINING COURSE CA1 "CORE APPLICATIONS CONCEPTS"

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Following the rules of British Faculty and Institute of Actuaries the **Diploma in Actuarial Techniques** will be sent directly to students completing all of the Core Technical stage subjects: CT1, CT2, CT3, CT4, CT5, CT6, CT7, CT8 and CT9. The **Certificate in Finance and Investment** is a joint certificate and will be sent to all students of the Faculty and Institute of Actuaries, who complete or are exempted from CT1, CT2, CT4, CT7, CT8, CT9 and CA1.

At the Training Centre for Actuaries and Financial Analysts on the base of Kyiv National Taras Shevchenko University the courses from Core Technical stage CT1 – CT8 of British Examination System was developed in a frame of European Project TEMPUS IB-JEP-25054-2004 and are available for practitioners from insurance industry. All aforementioned courses are delivered by professors and lecturers of Faculty of Mechanics and Mathematics.

In this presentation the content of training course CA1 "Core Applications Concepts" for actuaries within the British Examination System will be discussed.

РОЛЬ І ОБОВЯЗКИ АКТУАРІЯ У НОВІЙ РЕДАКЦІЇ ЗАКОНУ УКРАЇНИ «ПРО СТРАХУВАННЯ»

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У доповіді розглядається нова редакція Закону України «Про страхування». Зокрема обговорюється питання актуарного забезпечення страхової діяльності, передбаченого новою редакцією Закону.

TRENDS IN, AND REASONS FOR, REORGANIZATION OF UNIVERSITIES – A SWEDISH CASE

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During the last ten years there have been several reorganizations at Swedish universities and also plans for merging and federations between them. Similarities can be found in other countries in Western Europe. Some international examples will be demonstrated. Often these changes have political backgrounds and the action is summarized in three words: profiling, cooperation and concentration.

I will give a short resume of how Swedish universities traditionally have been organized, and the political reasons for the resent change and how these changes affect the activities at the universities. Also, I will speculate what this means for the future.

An example will also be given from Mälardalen University how such reorganization can be initialized, planed and conducted. Is there a lesson to be learned?

RISK MODELS WITH EXTREMAL SUBEXPONENTIALITY

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In this paper we consider risk models with a heavy-tailed parametric claim distribution from the subexponential class S with at least two parameters. We choose a proper convergence of a parameter, that makes the tail of the claims distribution heavier or lighter and then tend it to its limitation. Finally we proceed to an appropriate functional normalization in order to keep the distributional properties.

STORAGE PROCESSES IN POISSON APPROXIMATION SCHEME.

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Discrete storage processes, given by a sum of random variables on Markov and semi-Markov processes, are approximated by the Poisson compound processes on increasing time intervals.

ON CONVERGENCE OF WAVELET EXPANSIONS OF RANDOM PROCESSES

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In the paper we present conditions for convergence in mean square and uniform convergence in probability on [0,T] of wavelet expansions of random process $X = \{X(t),\, t \in R\}$ with $\mathrm{EX(t)=0,\ E|X(t)|^2<\infty}$. We obtained the rate of convergence of wavelet representation of random processes in the norm of the space $\mathrm{C(0,T)}$ as well.

Let $\varphi_{jk}(x)=2^{j/2}\varphi_{jk}\left(2^j x\text{-k}\right)$ and $\psi_{jk}(x)=2^{j/2}\psi_{jk}\left(2^j x\text{-k}\right)$ **(1)**

We consider wavelet representations of random processes:

$$X(\tau)=\sum_{k\in Z}\alpha_{0k}\varphi_{0k}(\tau)+\sum_{j=0}^{\infty}\sum_{k\in Z}\beta_{jk}\psi_{jk}(\tau) \qquad \textbf{(2)}$$

$$X_n(\tau)=\sum_{k\in Z}\alpha_{0k}\varphi_{0k}(\tau)+\sum_{j=0}^{n-1}\sum_{k\in Z}\beta_{jk}\psi_{jk}(\tau) \qquad \textbf{(3)}$$

$$X_{n,k_j}(\tau)=\sum_{|k|\le k_0}\alpha_{0k}\varphi_{0k}(\tau)+\sum_{j=0}^{n-1}\sum_{|k|\le k_j}\beta_{jk}\psi_{jk}(\tau) \qquad \textbf{(4)}$$

where $\alpha_{0k}=\int_R X(\tau)\overline{\varphi_{0k}(\tau)}d\tau\,,\ \ \beta_{jk}=\int_R X(\tau)\overline{\psi_{jk}(\tau)}d\tau$ **(5)**

Theorem 1. Let $X = \{X(t),\, t \in R\}$ be a random process such that $\mathrm{EX(t)=0,\ E|X(t)|^2<\infty}$ for all $t \in R$ with continuous covariance function $\mathrm{R(t,s)}$. Let wavelets $\varphi(x)$ and $\psi(x)$ be continuous functions. Let the assumption S hold true for an f-wavelet $\varphi(x)$ and m-wavelet $\psi(x)$, $c = \{c(x),\, x \in R\}$ be such an even function, that $c(x) > 1, x \in R$ and non decreases as $x>0$, and there exists such function $0<A(a)<\infty, a>0$, that for sufficiently large x: $c(ax) \leq c(x) \cdot A(a)$. If $\int_R c(x)\Phi(|x|)dx < \infty$ and $|R(t,t)|^{1/2} \leq c(t)$ then

1) $X_n(t) \in L_2(\Omega)$ and $X_{n,k_j}(t) \in L_2(\Omega)$

2) $X_n(t) \to X(t)$ as $n \to \infty$ in mean square.

3) $X_{n,k_j}(t) \to X(t)$ as $n \to \infty, k_j \to \infty, \forall j=0,1,...$ in mean square.

Theorem 2. Let $X = \{X(t), t \in R\}$ be a centered random process such that $EX(t)=0$, $E|X(t)|^2<\infty$ for all $t \in R$ with continuous covariance function $R(t,s) = \int_R f(t,\lambda)\overline{f(s,\lambda)}d\lambda$, $\int_R |f(t,\lambda)|^2 d\lambda < \infty$. Let the assumptions of Theorem 1. hold true. For an wavelets $\varphi(x)$ and $\psi(x)$ the following conditions hold:

1) $\exists \hat{\psi}^{(k+1)}(u)$, $\hat{\psi}^{(k)}(0)=0$, $k=\overline{0,2}$

2) $\exists c_{\varphi} = \sup_u |\hat{\varphi}(u)| < \infty$, $c_{\varphi'} = \sup_u |\hat{\varphi}'(u)| < \infty$, $c_{\varphi^{(3)}} = \sup_u |\hat{\varphi}^{(3)}(u)| < \infty$

3) $\hat{\varphi}(u) \to 0$, $u \to \pm\infty$, $\hat{\psi}(u) \to 0$, $u \to \pm\infty$

4) $\exists \int_R |\hat{\varphi}(u)| u^{\beta} du < \infty$, $\int_R |\hat{\varphi}'(u)| u^{\beta} du < \infty$,

$\int_R |\hat{\psi}(u)| u^{\beta} du < \infty$, $\int_R |\hat{\psi}'(u)| u^{\beta} du < \infty$, $\int_R |\hat{\psi}''(u)| u^{\beta} du < \infty$, $1/2 < \beta < 1$,

5) $\exists \int_R \left| \frac{\partial^k f(z,\lambda)}{\partial z^k} \right| |z|^p dz < \infty$, $k = 0,1$; $p = \overline{0,3}$,

6) $|f(z,\lambda)| < a < \infty$.

Then $X_{n,k_j}(t) \to X(t)$ as $n \to \infty, k_j \to \infty, \forall j=0,1,...$ uniformly in probability on each interval [0,T]

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RESELLING OF EUROPEAN OPTIONS

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European options can only be exercised at maturity. There however exist a possibility of selling the option at time *t* on the open market and receive the market price C_t^m. We propose a continuous model of the problem and in order to develop ε-optimal strategies we also propose a discrete approximation, and show that the approximation converge to the model. The discrete model allows us to use the method of backward induction to find the optimal reselling time and to study the structure of the stopping domain.

QUANTILE HEDGING WITH REDISCOUNTING ON THE COMPLETE FINANCIAL MARKET

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The talk is devoted to the problem of quantile hedging of contingent claims in the framework of a mixed Brownian-fractional-Brownian motion model. We follow the approach of Fellmer and Leukert but modify their approach since if we go by direct way, then in the model with long-range dependence very complicated probability density functions appear. The main idea is to rediscount the original market and the contingent claim.

OPTIMAL TIME TO EXCHANGE FINANCIAL ASSETS ON A FINITE INTERVAL

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Let $S_1(t)$, $S_2(t)$ be correlated geometric Brownian motions. We consider the maximization problem $\mathrm{E}[S_1(\tau) - S_2(\tau)] \to \max$, where τ is a stopping time from $[0,T]$. This problem arises naturally from the problem of optimal exercise of a futures contract to exchange two correlated assets in the Black-Scholes market model. To solve this problem is crucial for purposes of strategic investment, which usually uses offsetting positions in correlated assets to hedge its losses.

A similar problem, but on infinite time interval, was studied in [2], and in [1], where a multi-asset generalization is also considered. For a finite time horizon, the problem gets considerably more complicated and cannot be solved explicitly. In this paper we study generic properties of the optimal stopping set and its boundary curve, and derive an integral equation for the latter.

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MINIMAX ESTIMATION PROBLEMS FOR CYCLOSTATIONARY DISCRETE TIME STATIONARY STOCHASTIC PROCESSES

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The problem of optimal linear estimation of the linear functional $A_P\chi = \sum_{j\in P} a(j)\chi(j)$ which depends on the unknown values of a periodically stationary (cyclostationary) discrete time stochastic process $\chi(j)$ from observations of the process $\chi(k)+\eta(k)$ for $k \in Z \setminus P$, where $\eta(j)$ is a periodically stationary process uncorrelated with $\chi(j)$. Formulas are proposed for calculation the mean square error and spectral characteristics of the optimal linear estimate in the case where spectral densities $f(\lambda)$, $g(\lambda)$ of processes $\chi(j)$, $\eta(j)$ are known. In the case where spectral densities are not known but special classes D_f and D_g of densities are given the least favourable spectral densities in these classes and the minimax-robust spectral characteristics of the optimal linear estimates of the functional $A_P\chi$ are found for some special classes of spectral densities.

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ПРОБЛЕМИ РОЗВИТКУ СТРАХОВИХ І ФІНАНСОВИХ РИНКІВ НА УКРАЇНІ

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Дається характеристика сучасного стану і головних проблем розвитку фінансових і страхових ринків України, включаючи фондовий ринок, ринок банківських послуг, ринків особистого і ризикового страхування. Значна увага приділяється стану підготовки кваліфікованих фахівців в галузях аналітичних фінансів і фінансового менеджменту, страхової справи та актуарних розрахунків. Проводиться огляд перспектив та прогнозів розвитку страхових і фінансових ринків України.

OPTIMAL PRICING OF AMERICAN TYPE OPTIONS FOR MODULATED PRICE PROCESSES

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This lecture presents a survey of the latest results on option optimal pricing for modulated price processes achieved by the author and his collaborators. These results are: discovery of multi-threshold structure of optimal stopping strategies for option models with general convex payoffs and formulation of conditions, which implicate multi- and one-threshold structures for optimal stopping strategies; introduction and investigation of new models of pricing processes modulated by semi-Markov market indices; obtaining of skeleton approximations, uniform with respect to a perturbation parameter, for continuous- and discrete-time option pricing models; finding of new effective general conditions for convergence of option reward functions; constraction of effective Monte Carlo algorithms for pricing of options based on information about structure of optimal stopping domains, experimental software for pricing of options, and the latest achievements are connected with stochastic models for reselling of options.

CHANGE POINT ANALYSIS OF EXTREME VALUES

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In a sample from the distribution of a random variable, it is possible that the tail behavior of the distribution changes at some point in the sample. This tail behavior can be described by absolute or relative excesses of the data over a high threshold, given that the random variable exceeds the threshold. The limit distribution of the absolute excesses is given by a Generalized Pareto Distribution with an extremal parameter γ and a scale parameter σ. When the extreme value index γ is positive, then the relative excesses can be described in the limit by a Pareto distribution with this index as parameter. In this lecture we concentrate on testing whether changes occur in the value of the extreme value index γ and/or the scale parameter. To this end, appropriate test statistics are introduced based on the likelihood approach of Csörgő and Horváth (1997) for independent data. Asymptotic properties of these test statistics lead to adequate critical values. After giving a practical test procedure, we apply our results to a series of simulations and real life examples, some related to business and industry.

ON BELONGING OF A STOCHASTIC PROCESS TRAJECTORIES TO A SOBOLEV-ORLICZ SPACE

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Basic notions on the Orlicz space theory will be presented. A stochastic process will be investigated within this theory. Conditions under which a stochastic process trajectory belongs with probability one to the Orlicz space or to the Sobolev-Orlicz space will be studied. As a particular case, we will consider a stochastic process in a functional $L_q(R)$-space and in a classical Sobolev space $W_q^N(R)$.

References.

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SOME APPLICATIONS OF Φ-SUB-GAUSSIAN RANDOM PROCESSES

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Investigation of the classes of φ-sub-Gaussian and strictly φ-sub-Gaussian random processes is an actual task of modern theory of stochastic processes since such classes of random processes are more general than Gaussian one and also contains processes with heavier or lighter tails than tails of Gaussian processes. More details on the spaces of φ-sub-Gaussian random variables and processes can be found in [1],[2].

We focus on estimating of the probability of overrunning by trajectories of a φ-sub-Gaussian random process a level specified by some continuous function (see also [3]). This type of problem often arises in various applications in financial mathematics, risk theory, queuing theory, etc. We continue that investigation and apply it to the processes of generalized φ-sub-Gaussian fractional Brownian motion, the processes that have the same covariance function as fractional Brownian motion but whose trajectories are φ-sub-Gaussian, i.e. not necessarily Gaussian (see [4]). As examples several applications of such generalized processes are given.

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FLUCTUATIONS OF THE TOTAL CLAIM AMOUNT PROCESS

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Let S(n) be the sum of n independent identically distributed random variables, N(t) – the renewal counting process, D(t) =S(N(t)) – randomly stopped sums. We study the asymptotic behaviour of D(t) and its increments of the type d(t)= D(t + a(t)) - D(t) over the intervals whose length a(t) grows as t grows, but not faster then t. Modifications of the LIL and Erdos-Renyi strong law of large numbers are proved under various assumptions on summands and renewal process. The main tool of investigation is the strong invariance principle for the superposition of random processes. Obtained results can be used for investigation of fluctuation of the total claim amount process in the collective Sparre Andersen risk model.

CONTRIBUTED LECTURES

THE MODEL OF LIFE INSURANCE COMPANY BASED ON STOCHASTIC MACRO-ECONOMICAL INDEXES.

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Main object of investigation is life insurance company's activity. The purpose of research – construction the adequate model of company's functioning which give the possibility to describe and predict the behavior of the main economic indexes of business: the amounts and times of necessary future financial investments, results of insurance, financial, investment, management activity, required sum of mathematical reserves and a lot of important characteristics for insurance. Stochastic basis of the model – discrete hierarchical approach for modeling the main macro-economical indexes (inflation rate, prices and yields of bonds, shares, precious metals, deposits and properties), that affect the development of company markedly. This is the original model, which shows the authors' point of view on the nature of stochastic processes in life insurance and its influence on the activity of the company.

SOLVENCY OF AN INSURANCE COMPANY IN DISCRETE MODEL

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Let $T_1, T_2, \ldots,$ be a sequence of mutually independent random variables, the inter-arrival times, with a common generating function $g_T(z) = \sum_{k=1}^{\infty} q_k z^k$ and mean value $ET_k = 1/\alpha$, $Z_1, Z_2, \ldots,$ be a sequence of mutually independent random variables, the claims to an insurance company, with a common generating function $g_Z(z) = \sum_{k=1}^{\infty} p_k z^k$ and mean value $EZ_k = \mu$, and $c > \alpha\mu$ be the gross premium rate, $c \in \mathbf{N}$. The non-ruin probabilities, φ_u, of the insurance company with initial capital $u \in \mathbf{Z}$ satisfies the infinite system of linear algebraic equations of Wiener-Hopf type,

$$\varphi_u = \sum_{v=1}^{\infty} q_v \sum_{z=1}^{u+cv} \varphi_{u+cv-z} p_z \,, \quad \varphi_{-\infty} = 0 \,, \quad \varphi_{\infty} = 1 \,, \qquad (1)$$

that is the discrete analog of the Lundberg-Feller equation [1,2]. Note that the initial capital u of the company can accept both non-negative and negative integers. The non-ruin probability φ_u in the latter case means the probability that the company begins the activity having a debt $(-u)$, however to the moment of the first claim the capital of the company becomes positive and is hereinafter not below of zero. In the case when the random variables have the shifted geometrical distribution with the generating function

$$g_T(z) = \sum_{n=1}^{\infty} p(1-p)^{n-1} z^n \,, \qquad 0 < p < 1, \qquad (2)$$

the problem (1) was considered with the algebraic method by A.Melnikov [3].

In this work for the solution of the problem (1) the discrete analogue of the Wiener-Hopf method [4] is applied that makes possible to obtain the solution of the problem in the very general case. We shall seek the solution of the problem in the form of the generalized generating function $\varphi(t)$ defined on the unit circle $L = \{t : |t| = 1\}$:

$$\varphi(t) = \varphi^+(t) - \varphi^-(t)\,, \qquad t \in L\,,$$

where the series

$$\varphi^+(z) = \sum_{n=0}^{+\infty} \varphi_n z^n \,, \qquad -\varphi^-(z) = \sum_{n=-1}^{-\infty} \varphi_n z^n$$

converge for $|z| < 1$ and $|z| > 1$, respectively.

Going over to the generating functions in (1), we obtain the Riemann boundary value problem

$$\varphi^+(t) - \varphi^-(t) = g_T(t^{-c}) g_Z(t) \varphi^+(t)\,, \qquad t \in L\,. \quad (3)$$

The symbol of the equation (1), calculated by the formula

$$A(t) = 1 - g_T(t^{-c}) \cdot g_Z(t)\,, \qquad t \in L\,,$$

is differentiable function on L, certainly has zero of the first order at the point $t = 1$ and has not other singularities on L, and also satisfies the condition

$$A(\bar{t}) = \overline{A(t)}\,, \qquad t \in L\,.$$

The solution of the problem (1) is obtained from the factorization of the symbol $A(t)$, which has the form

$$A(t) = (1-t) A^+(t) t^{-1} A^-(t)\,, \qquad t \in L\,, \qquad (4)$$

where $A^+(z) \neq 0$, $|z| < 1$, $A^-(z) \neq 0$, $|z| > 1$, $A^+(1) = 1$, $A^-(1) = \frac{c}{\alpha} - \mu$. Using the results of the monograph [4], from this factorization follows that the equation (1) has two linear independent solutions from which one is irrelevant and the second one is the required solution which has the form

$$\varphi^+(t) = \frac{1}{(1-t)A^+(t)}\,, \qquad \varphi^-(t) = t^{-1} A^-(t)\,, \qquad t \in L\,.$$

Expanding the functions $\varphi^+(z)$ and $\varphi^-(z)$ into a power series about z i $1/z$, respectively, we obtain required probabilities φ_n.

The delay S_0 for the delayed stationary renewal epochs $S_n = S_0 + T_1 + ... + T_n$ has the generating function

$$g_{S_0}(t) = \frac{\alpha \cdot t \cdot (1 - g_T(t))}{1-t}\,, \qquad t \in L\,. \qquad (5)$$

Then the generating function $\varphi^s(t)$ for non-ruin probabilities in the delayed stationary process has the form

$$\varphi^s(t) = \varphi_s^+(t) + \varphi_s^-(t) = g_{S_0}(t^{-c}) \cdot g_Z(t) \cdot \varphi^+(t)\,, \quad t \in L\,, \quad (6)$$

whence the probabilities φ_n^s are obtained by the decomposition of the functions $\varphi_s^+(z)$ and $\varphi_s^-(z)$ into a power series about z i $1/z$, respectively.

Using (3) and (5) we can simplify the formula (6) excepting the generating function $g_T(t)$:

$$\varphi^s(t) = \frac{\alpha(1 - g_Z(t^{-c}))\varphi^+(t) - \alpha\varphi^-(t)}{1 - t^c}, \qquad t \in L. \qquad (7)$$

In the case when $c = 1$, by projecting the equation (7) on the space of the functions $\varphi(z)$ holomorphic outside of L, for which $\varphi(\infty) = 0$, we obtain the formula $\varphi_{-1}^s = \alpha\varphi^-(1)$, whence $\varphi_{-1}^s = 1 - \alpha\mu$. Projecting then the equation (7) on the space of the functions $\varphi(z)$ holomorphic inside of L, we obtain the formula:

$$\overset{s}{\varphi}(z) = \frac{\overset{s}{\varphi}_{-1}}{z} + \frac{\alpha(1 - g_Z(z))\varphi^+(z)}{1 - z}, \quad |z| < 1.$$

In the cases when T_k have rational generating functions (shifted geometrical, binomial or negative binomial with positive integer degree, etc), the symbol $A(t)$ of the equation (1) can be easily factored and the solution of the problem (1) can be obtained in explicit form. Number of illustrative examples is considered.

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APPROXIMATION OF RANDOM PROCESSES BY SPLINES

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Approximation and reconstruction of random processes are one of important problems in the theory of stochastic processes as in real life any stochastic process can be observed only in a finite number of moments of time.
The approximation of random processes is applied in different branches of nature and social sciences, specifically in financial and actuarial mathematics etc.
New results which may be used to restore sub-Gaussian process by cubic splines in different spaces with given accuracy and reliability are obtained.

Definition 1. The process $\overset{\Box}{X}(t)$ approximates stochastic process $X(t)$ with given accuracy $\varepsilon > 0$ and reliability $1-\delta, \delta \in (0,1)$ in the space A if $P\{\| \overset{\Box}{X}(t) - X(t) \|_A > \varepsilon\} \leq \delta.$

Denote $\Delta := \{a = t_0 < \ldots < t_N = b\}$ – partition of the segment $[a,b]$, $\|\Delta\| = \max\limits_{i=\overline{0,N-1}} (t_{i+1} - t_i),\ \beta : \max\limits_{i=\overline{0,N-1}} \dfrac{\|\Delta\|}{t_{i+1} - t_i} \leq \beta < \infty.$

Definition 2. $S_\Delta(t),$ continuous on $[a,b]$ with its first and second derivatives, which is a cubic polynomial on every segment $[t_i, t_{i+1}], i = \overline{0, N-1},$ and satisfies $S_\Delta(t_i) = X(t_i), i = \overline{0, N-1},$ is called a cubic spline on Δ which interpolates $X(t)$.

The following theorems hold true.

Theorem 1. Let $X = \{X(t), t \in T\}$ be a strictly sub-Gaussian random process and assume $\sup\limits_{t\in[a,b]} E\left|X(t+h) - X(t)\right|^2 \leq b^2(h),$ where $b(h), h > 0$ is a positive monotonically increasing function, $\dfrac{h}{b(h)}, h > 0$ is non-decreasing and $b(h) \downarrow 0, h > 0.$

Then the following inequality holds:

$$\sup_{t\in[a,b]} \left(E\left|S_\Delta(t) - X(t)\right|^2 \right)^{1/2} \leq \left(\frac{3}{2} + \frac{20}{27}\beta^2 \right) b(\|\Delta\|). \qquad (1)$$

Theorem 2. Assume the conditions of the theorem 1 hold and denote $Y_N(t) := S_\Delta(t) - X(t), t \in [a,b]$ – deviation process. Then $\forall h > 0$ the following estimate holds:

$$\sup_{t\in[a,b]} \left(E\left|Y_N(t+h) - Y_N(t)\right|^2 \right)^{1/2} \leq \left(4 + 8\beta^3\right) b(h). \qquad (2)$$

R e m a r k 1. Inequalities (1) and (2) allow us to approximate different classes of $SSub(\Omega)$ random processes with given accuracy and reliability in the norms of spaces $L_p([a,b]), C([a,b])$ etc.

R e m a r k 2. Similar results can be obtained for strictly $\varphi-$sub-Gaussian random processes.

"СТАТИСТИКА" – ЯК НОВИЙ НАПРЯМ МАТЕМАТИЧНОЇ ОСВІТИ В УКРАЇНІ. ПРО КОНЦЕПЦІЮ ПІДГОТОВКИ ФАХІВЦІВ -СТАТИСТИКІВ

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Із вступом України на шлях ринкових економічних відносин виникла потреба у висококваліфікованих спеціалістах нового профілю, які раніше не готувались нашими вузами. Однією із таких спеціальностей є ***статистика*** в сучасному розумінні, яка була відсутня в державній номенклатурі спеціальностей. В той же час дана спеціальність є однією з провідних у всіх університетах Європи, Америки, Азії.

Спеціальність ***"статистика"*** включає в себе підготовку фахівців з наступних напрямків:

- математична статистика;
- математична економіка та економетрика;
- фінансова та актуарна математика;
- аналітичний менеджмент;
- теорія ризику, теорія прогнозування та стратегічного планування;
- сучасна економіко-фінансова та соціальна статистика тощо.

Ринкова економіка — це економіка постійних багатократних виборів в умовах різних невизначеностей та економічного, ділового і фінансового ризиків. Стан речей в сучасній економіці, фінансах, страхуванні вимагає від фахівців крім економічних знань ще й ґрунтовного володіння складним аналітичним та математичним апаратами.

За сучасними дослідженнями біля 85% наукових праць з економіко-фінансових і страхових дисциплін мають справу із складними математико-статистичними моделями і приблизно така ж кількість Нобелівських лауреатів з економіки є професійними математиками та статистиками.

Як свідчить світова практика, на математичних факультетах, де викладаються розвинуті курси математичних дисциплін, можна готувати спеціалістів, які творчо володіють теоретико-ймовірносними та статистичними методами і вміють їх застосовувати до розв'язання складних проблем соціальних наук, економіки, фінансів, тощо.

Механіко-математичний факультет протягом кількох років вивчав світовий досвід і самостійно почав здійснювати підготовку спеціалістів із вказаних напрямів. На основі аналізу учбових планів ряду університетів США, Франції, Росії, Польщі, Австрії, України створено власний навчальний план підготовки, **який відрізняється від навчального плану підготовки спеціальності "Математика".**

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ГРАВІТАЦІЙНІ МОДЕЛІ ОПТИМІЗАЦІЇ ФІНАНСОВОЇ МОДЕЛІ ДІЯЛЬНОСТІ БАНКУ

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Побудовано гравітаційну модель активно-пасивних операцій. Як результат знайдено величину оптимального процентного доходу від здійснення активно-пасивних операцій.

Зазначимо, що розглянута модель дозволяє врахувати імовірнісний характер отримання результатів від проведення фінансово-кредитної діяльності комерційним банком. Зокрема, під час формування вхідних даних, які відображають схему діяльності банку враховано по перше – два типи ризиків успішного здійснення активно–пасивних операцій, по друге коефіцієнт планової зміни розміру залучених та вкладених коштів. Зазвичай, вказані величини визначаються групою експертів банку на основі певних аналітичних досліджень. Далі, використання математичної моделі, дає можливість поєднати формальні та неформальні способи ефективного ведення фінансової діяльності банком. Вказаний підхід дозволяє значною мірою наблизити змодельований процес планування роботи банку до реально діючих методик та максимізувати фінансовий дохід від операційної діяльності.

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MODELLING OF RANDOM PROCESS WITH KNOWN CORRELATION FUNCTION WITH THE HELP OF KARHUNEN-LOEVE DECOMPOSITION

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Gaussian processes are good in describing many problems in financial and actuarial mathematics. Modelling of Gaussian processes has many applications in this field.

For a centred Gaussian stochastic process with the known correlation function we consider the Karhunen-Loeve decomposition of the process, which represents the process using eigenfunctions and eigenvalues of some integral equations. We are to build a Karhunen-Loeve model of the process with known correlation function that approximates this process with given reliability and accuracy.

Little integral equations types can be solved in an explicit form. That is the main problem while building a Karhunen-Loeve model of stochastic process. We use approximations of eigenfunctions and eigenvalues and construct a theorem that allows us to build a Karhunen-Loeve model of stochastic process with given reliability and accuracy even if we don't know the Karhunen-Loeve decomposition of this process.

Algorithms of modelling and estimation are created using Mathematica 6.0 software.

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MULTIVARIATE SECOND ORDER RANDOM FIELDS OVER SOME HOMOGENEOUS SPACES

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In the theory of second order random functions (processes and fields) and its statistical applications the central role plays the spectral and related representations of such functions and their covariances. This communication is devoted to spectral theory of multivariate second order random fields over homogeneous space X with compact transitive transformation groups for the case in which the value of such fields are generalized random elements of second order in arbitrary normed and Hilbert spaces. It is proved that all the continuous multivariate in such sense random fields order X are harmonizable. The spectral representations of weakly homogeneous multivariate random fields order X are obtained. These results are specialized foe isotropic random fields over spheres in R and R.

ОЦЕНКА ВЕРОЯТНОСТИ ХВОСТА ДВОЙНОГО МАКСИМУМА ГАУСОВСКОГО ПРОЦЕССА

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В математической статистике задача построения доверительного интервала для неизвестной плотности вероятности с помощью эмпирической плотности распределения приводится к нахождению предельного распределения максимума гауссовского нестационарного процесса, эта задача связана с исследованием распределения максимума гауссовского нестационарного процесса и оценкой распределения максимума неоднородного поля. В данной работе оценивается распроделение двойного максимума гаусовского процесса.

Пусть $X(t), t \in T \subseteq R$ -сепарабельный гауссовский процесс со средним нуль и дисперсией единица, имеющий при всех t среднеквадратичную производную $X'(t)$.Обозначим:

$$\rho(t,s) = \mathrm{cov}(X(t), X(s)), \quad \beta(t) = \mathrm{var}X'(t), \qquad B_T = \sup_{t\in T} \beta(t)$$

Доказаны:

ТЕОРЕМА 1. Пусть $0 < B_T < \infty$ и при $i \neq j$ замкнутые интервалы I_i, I_j содержатся в T и не пересекаются. Тогда при $i \neq j$ и для всех $u > 1$

$$P(\max_{t\in I_i} X(t) > u, \quad \max_{s\in I_j} X(s) > u) \leq C\lambda(I_i)\lambda(I_j)\frac{B_T}{\bar{\sigma}_+} u e^{-\frac{su^2}{\tilde{\sigma}_+^2}},$$

где C - положительная абсолютная постоянная, $\lambda(I)$ - лебегова мера интервала I,

$$\tilde{\sigma}_+^2 = \tilde{\sigma}_+^2(i,j) = 2\sup_{t\in I_i, s\in I_j}(1+\rho(t,s)), \quad \bar{\sigma}_+^2 = \bar{\sigma}_+^2(i,j)\inf_{t\in I_i, s\in I_j}(1+\rho(t,s)).$$

Введем обозначения:

$$\tilde{\sigma}_-^2 = \tilde{\sigma}_-^2(i,j) = 2\sup_{t\in I_i, s\in I_j}(1-\rho(t,s)), \quad \bar{\sigma}_-^2 = \bar{\sigma}_-^2(i,j)\inf_{t\in I_i, s\in I_j}(1-\rho(t,s))$$

Тогда имеет место следующая теорема:

ТЕОРЕМА 2. При условии теоремы 1 и для всех $u \geq 1$ справедливо неравенство

$$P(\max_{t\in I_i} X(t) > u, \quad \max_{s\in I_j}(-X(s)) > u) \leq C\lambda(I_i)\lambda(I_j)\frac{B_T}{\bar{\sigma}_-} u e^{-\frac{su^2}{\tilde{\sigma}_-^2}},$$

где C - положительная абсолютная постоянная, $\lambda(I)$ - лебегова мера интервала I