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TEMPUS PROJECT IB-JEP-25054-2004
Training Centre for Actuaries and Financial Analysts

ABSTRACTS

INTERNATIONAL SUMMER SCHOOL
"INSURANCE AND FINANCE: SCIENCE, PRACTICE AND EDUCATION"

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Organizers:

Kyiv Taras Shevchenko National University (Ukraine)
Mälardalen University (Sweden)
Training Centre for Actuaries and Financial Analysts
Society of Actuaries of Ukraine
State Commission for Regulation of the Financial Services Markets of Ukraine

Organizing Commettee:

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Dmitrii Silvestrov (Västerås, Sweden)- co-chairman
Oleksandr Borysenko (Kyiv, Ukraine)- secretary
Yurii Ivanko (Kyiv, Ukraine)
Evelina Silvestrova (Västerås, Sweden)
Nadija Zinchenko (Kyiv, Ukraine)
Galina Bagro (Kyiv, Ukraine)

International Summer School "**Insurance and Finance: Science, Practice and Education"** is held under the support of the EU within the framework of the EU TEMPUS PROJECT IB-JEP-25054-2004 "Training Centre for Actuaries and Financial Analysts "

Main topics:

- Actuarial Sciences
- Analytical Methods in Finance
- Educational Actuarial Programs
- Training Center for Actuaries and Financial Analysts
- Problems of Insurance Business in Ukraine and Actuarial Support of Insurance Companies Activity.

INVITED LECTURES

RUIN PROBABILITIES FOR INSURANCE MODELS INVOLVING INVESTMENTS

Maryna Androshchuk
Kyiv National Taras Shevchenko University
Kyiv, Ukraine
E-mail: andr_m@univ.kiev.ua

We study the problem of ultimate ruin probability estimation for an insurance company which invests into some stock market. The market is supposed to contain a finite number of different shares, which are modeled by geometrical Brownian motions, possibly correlated, and also a current account.

Non-classical feature of our risk model is the form of premium rate process which is allowed to vary together with the value of the current capital of insurance company.

We obtain Cramer-Lundberg type estimations of ruin probability using supermartingale technique ([1], [2]). In the case of particular constant investments into every share we get exponential estimate which presents sharper bound of the ruin probability comparing with classical case.

The results obtained generalize also result of the paper [3], in which the estimation of ruin probability for the model with constant premium rate and investments in one type of shares was established.

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MODELS OF MORTALITY IN ACTUARIAL TRAINING COURSE "MODELS"

Oleksandr Borysenko
Kyiv Taras Shevchenko National University
E-mail: odb@univ.kiev.ua

The content of training course "Models" for actuaries within the British Examination System will be discussed. Description of the two-state Markov model, the general Markov model, Binomial and Poisson model of mortality will be presented.

НОВА РЕДАКЦІЯ ЗАКОНУ УКРАЇНИ «ПРО СТРАХУВАННЯ»

Бутова Віра Василівна
Головний спеціаліст відділу методології недержавного пенсійного забезпечення та страхування життя

У доповіді розглядається нова редакція Закону України «Про страхування». Зокрема обговорюється питання актуарного забезпечення страхової діяльності, передбаченого новою редакцією Закону.

RISK MODELS WITH EXTREMAL SUBEXPONENTIALITY

D.G. Konstantinides
Department of Statistics and
Actuarial - Financial Mathematics,
University of the Aegean, Karlovassi,
GR-83 200 Samos, Greece
E-mail address: konstant@aegean.gr

In this paper we consider risk models with a heavy-tailed parametric claim distribution from the subexponential class S with at least two parameters. We choose a proper convergence of a parameter, that makes the tail of the claims distribution heavier or lighter and then tend it to its limitation. Finally we proceed to an appropriate functional normalization in order to keep the distributional properties.

LINEAR INTERPOLATION OF RANDOM PROCESSES IN THE SPACE $L_p(T)$

Yu. V. Kozachenko, O. Y. Kamenschykova
Kyiv National Taras Shevchenko University
yvk@univ.kiev.ua, palamar_a@mail.ru

One of important problems of the theory of stochastic processes is approximation and reconstruction of stochastic processes. Many methods of stochastic simulation are investigated to date. The approximation of random processes is applied in different branches of nature and social sciences, in financial and actuarial mathematics etc.

New inequality which may be used to construct the broken line which restores the given sub-Gaussian process in the space $L_p(T)$ is obtained.

Let (T, A, μ) be a measurable space with σ-additive measure μ. The following theorem holds true.

Theorem 1. Let $X = \{X(t), t \in T\}$ be a sub-Gaussian random process and assume there exists the Lebesque integral $\int_T \tau^p(X(t))d\mu(t)$. Then for all $\varepsilon > p^{p/2}\int_T \tau^p(X(t))d\mu(t)$ the next inequality holds

$$P\{\int_T |X(t)|^p d\mu(t) > \varepsilon\} \le 2\exp\left\{-\frac{\varepsilon^{2/p}}{2(\int_T \tau^p(X(t))d\mu(t))^{2/p}}\right\}. \quad (1)$$

R e m a r k. For Gaussian random process „2” before “exp” may be replaced by $\sqrt{2}$ in (1).

Definition 1. The process $X_N(t)$ approximates stochastic process $X(t)$ with given accuracy $\varepsilon > 0$ and reliability $1-\delta$, $\delta \in (0,1)$ in the space $L_p(T)$ if

$$P\{\| X_N(t) - X(t) \|_{L_p} > \varepsilon\} \le \delta.$$

BOUNDS FOR A SUM OF RANDOM VARIABLES UNDER A MIXTURE OF NORMALS

Alexander Kukush and Mychailo Pupashenko
Kyiv National Taras Shevchenko University

In an insurance context, one is often interested in the distribution function of a sum of random variables (rv's). Such a sum appears when considering the aggregate claims of an insurance portfolio over a certain reference period. It also appears when considering discounted payments related to a single policy or a portfolio, at different future points in time. The assumption of mutual independence of the components of the sum is very convenient from a computational point of view, but sometimes not a realistic one. In the papers Dhaene et al. (2002a) and (2002b) the approximations for sums of rv's were derived when the distributions of the components are known, but the stochastic dependence structure is unknown or too cumbersome to work with. In this paper we consider the case of a switching regime which can represent a change in the economic environment, see Yang (2006).

We deal with the problem similar to presented in Dhaene et al. (2002b), Section 4.1. Let $Y_1,\ldots,Y_n$ be a sequence of identically distributed rv's. Consider the sum $S := \sum_{i=1}^{n} \alpha_i \exp(-(Y_1+\ldots+Y_i))$, where α_i are the scalars of any sign. Based on the comonotonicity approach, we give the upper and lower convex stochastic bound of S for the next two cases.

1. Y_j are independent with cumulative d.f. $\sum_{i=1}^{s} \pi_i N(\mu_i,\sigma_i^2)$, where normals are independent and $\pi_i > 0, i=\overline{1,s},\ (\mu_i,\sigma_i^2) \neq (\mu_p,\sigma_p^2), i \neq p.$
2. $Y_j = Y_j^{\xi_j}, j=\overline{1,n}$, where $\{\xi_1,\ldots,\xi_n\}$ - is a finite-state, time-homogeneous Markov Chain, with space $S=\{1,\ldots,s\}$. The conditional distribution law of Y_j given ξ_j equals $L(Y_j^{\xi_j} \mid \xi_j = k) = L(Y_j^k) = N(\mu_k,\sigma_k^2), k=\overline{1,s},\ j=\overline{1,n},$ where $(\mu_i,\sigma_i^2) \neq (\mu_p,\sigma_p^2), i \neq p.$ The rv's $Y_j^k, j=\overline{1,n},\ \ k=\overline{1,s}$ are independent.

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STRUCTURE OF OPTIMAL STOPPING DOMAINS FOR AMERICAN PUT OPTIONS WITH KNOCK OUT DOMAINS.

Robin Lundgren
Mälardalen University
Västerås, Sweden
E-mail: robin.lundgren@mdh.se

American options give us the possibility to exercise them at any moment of time up to maturity. An optimal stopping domain for American type options is a domain that, if the underlying price process enters we should exercise the option.

A knock out option is a generalized barrier option of knock out type, but we let the domain generated by the barrier take any shape.

We extend the existing theory for discrete time optimal stopping rules, and the theory of generating optimal stopping domains.

An algorithm for generating the optimal stopping domain for American type knock out options is constructed. Monte Carlo simulation is used to determine the structure of the optimal stopping domain. We analyze the result and the stability of the solution. The results are presented for several combinations of payoff functions and knock out domains. This presentation also considers the probabilities of classification errors when determining the structure of the optimal stopping domain.

ROBUST ESTIMATION PROBLEMS FOR PERIODICALLY STATIONARY STOCHASTIC PROCESSES

Mikhail Moklyachuk
Kyiv Taras Shevchenko University,
Mechanics and Mathematics Faculty,
Kyiv 01033, Ukraine
e-mail: mmp@univ.kiev.ua

The problem of optimal linear estimation of the linear functional $A_P\xi = \sum_{j\in P} a(j)\xi(j)$ which depends on the unknown values of a periodically stationary (cyclostationary) discrete time stochastic process $\xi(j)$ from observations of the process $\xi(k)+\eta(k)$ for $k \in Z \setminus P$, where $\eta(j)$ is a periodically stationary process uncorrelated with $\xi(j)$. Formulas are proposed for calculation the mean square error and spectral characteristics of the optimal linear estimate in the case where spectral densities $f(\lambda)$, $g(\lambda)$ of processes $\xi(j)$, $\eta(j)$ are known. In the case where spectral densities are not known but special classes D_f and D_g of densities are given the least favourable spectral densities in these classes and the minimax-robust spectral characteristics of the optimal linear estimates of the functional $A_P\xi$ are found for some special classes of spectral densities.

SYSTEMS OF FINANCIAL ANALYSTS TRAINING

Mikhail Moklyachuk and Rostyslav Yamnenko
Kyiv National Taras Shevchenko University
mmp@univ.kiev.ua, yamnenko@univ.kiev.ua

We propose a review of different systems of financial analysts training which exist in European countries and the United States of America. MBA diploma and professional qualification such as Chartered Financial Analyst designation (CFA) in the United States of America, or Certified International Investment Analyst designation (CIIA) in Europe and Asia, are required for financial analysts to get certain level within a firm. We consider in details qualification levels that are offered by the most famous institutions such as:

- Faculty of Actuaries and Institute of Actuaries (UK);
- Chartered Financial Analyst Institute (USA);
- Association of Certified International Investment Analysts;
- Association of Corporate Treasurers (UK).

Curricula and corresponding exams are also reviewed.

COURSE "CONTINGENCIES" AT TRAINING CENTER FOR ACTUARIES AND FINANCIAL ANALYSTS

Andriy Olenko
Kyiv University, Ukraine
olenk@univ.kiev.ua

The course "Contingencies" taught at Training Center for actuaries and financial analysts (Kyiv University) was developed in a frame of European Project TEMPUS IB-JEP-25054-2004.

The aim of the course is to provide grounding in the mathematical techniques which are of particular relevance to actuarial work in life insurance, pensions, health and care. The course intends to give the students an insight into the life insurance business and its institutional organization. This course gives students basic knowledge of life insurance mathematics, mortality theory and more general stochastic processes in life insurance with applications to health insurance and multi-life insurance. Program of the course also includes such topics: the equivalence principle, prospective reserves and differential equations for these, administration costs, gross premiums and premium reserves. More practical topics on profit tests methods, unit-linked contracts, and pension benefits are taught in the second part of the course.

Internet cite of the course with lectures, problems, tables, a dictionary and another useful resources is developed. Problems book [1] and lectures are published.

The course is adapted to European standards in this area. One of the aims of this course is students' preparation to UK Faculty and Institute of Actuaries exams.

Description, aims and objectives of the course will be given in the presentation. Overview of the course's program, background readings and course Internet resources will be presented. Various aspects of future development and improving of the course will be discussed.

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ПРОБЛЕМИ РОЗВИТКУ ОСОБИСТОГО СТРАХУВАННЯ НА УКРАЇНІ

О.І.Пономаренко
Київський національний
університет ім. Тараса Шевченка
e-mail: probability@univ.kiev.ua

Дається характеристика сучасного стану особистого страхування на Україні, насамперед страхування життя та пенсійного страхування. Значна увага приділяється стану освіти в галузях страхової справи та актуарних розрахунків, а також відповідній класифікації працівників страхових компаній. Розглядаються основні проблеми, що виникли в цих сферах. Приводиться огляд перспектив та прогнозів розвитку особистого страхування на Україні.

RISK-MINIMIZING HEDGING IN THE MODEL WITH JUMPS

V.Radchenko
Kyiv National Taras Shevchenko University
e-mail: vradchenko@univ.kiev.ua

We consider the asset price process $\left(X_t\right)_{0\le t\le T}$ such that

$$X_t = X_0 + \int_0^t X_{s-}\left(a\,ds + \sigma dW_s\right) + \sum_{i:0<\delta_i\le t} X_{\delta_i -}U_{\delta_i} + \sum_{j:0<\tau_j\le t} X_{\tau_j -}U_{\tau_j}\ ,\ 0\le t\le T,$$

where $a\geq 0$ and $\sigma>0$ are constants, δ_i , $1\leq i\leq q$, are predictable stopping times and τ_j , $j\geq 1$, are jump times of homogeneous Poisson process.
For this model, the hedging problem is considered. The notions of mean-self-financing strategy and of risk-minimizing strategy are defined. We characterize the orthogonality of martingales as a property of risk minimality under small perturbations of stochastic integrals. Using this characterization, for finding of risk-minimizing strategy, a stochastic optimality equation is derived.

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ANALYTICAL FINANCE PACKAGE

Dmitrii Silvestrov
Mälardalen University
Västerås, Sweden
E-mail: dmitrii.silvestrov@mdh.se
Anatoliy Malyarenko
Mälardalen University
Västerås, Sweden
E-mail: anatoliy.malyarenko@mdh.se

Analytical Finance group at the Mälardalen University develops Analytical Finance Package (**AFP).** This is a library of Java applets and applications in the area of analytical finance. The project was initiated in 2003, and it is realising on a permanent base.

Professor Dmitrii Silvestrov and Dr. Anatoliy Malyarenko are leaders of the project. They develop **AFP** in collaboration with other members of Analytical Finance group and students of the Bachelor programme Analytical Finance and the Master programme Financial Engineering realising at the Mälardalen University. The names of developers of particular applets and applications are shown at their front pages.

The initial list of subject areas and number of items available now in each subject area are the following: A – Simulation of pricing processes (11 items); B – Estimation of pricing processes (0 items); C – Evaluation of financial contracts (22 items). The **AFP** is free software. The access to the program library **AFP** is available from the web-address: **http://www.mdh.se/ima/analyticalfinance/af1.software.shtml**. In order to make a browser adapted for running applets, users may need Java Runtime Environment which is downloadable from **http://java.sun.com/javase/downloads/index.jsp**.

ASYMPTOTIC EXPANSIONS FOR THE DISTRIBUTION OF THE SURPLUS PRIOR AND AT THE TIME OF A RUIN

Dmitrii Silvestrov
Mälardalen University
Västerås, Sweden
E-mail: dmitrii.silvestrov@mdh.se

Asymptotic expansions for the distribution of the surplus prior to and at the time of a ruin are obtained for a classical nonlinearly perturbed risk processes. Both cases of Cramér-Lundberg and diffusion approximations are considered in united form.

ON BELONGING OF THE STOCHASTIC PROCESS TRAJECTORIES TO SOME BESOV SPACES

Tetyana Yakovenko
Kyiv National Taras Shevchenko University
e-mail: yata452@univ.kiev.ua

We consider the stochastic process $X = \{X(t), t \in [a-\delta, b+\delta]\}$, $a < b$, $\delta > 0$ from space of random variables $L_p(\Omega)$. The main objective is to find conditions under which trajectories of this process belong with probability one to the Besov space $B_q^{\alpha p}[a,b]$ as $0 < \alpha < 1$ and $1 < p < q < \infty$ (Definition of the Besov spaces is taken from [1]). For this purpose we estimate the norm of increments of the stochastic process X and then its modulus of continuity using results obtained in [2, 3]. It gives the possibility to find the conditions for belonging the trajectories of the process to the Besov space with probability one.

The main result is the following theorem:

Theorem. If our stochastic process X is separable and measurable, $\sup_{t\in[a,b]}\left(E\,|\,X(t)\,|^p\right)^{1/p} < \infty$, and its increments satisfy two conditions:

a) $$\sup_{\substack{t\in[a,b],\\ |h|\le\delta}} \left(E\left|X(t+h)-X(t)\right|^p\right)^{1/p} \le C_p \delta^{\tau_p};$$

b) $$\sup_{\substack{t\in[a,b],\\ |h|\le\delta}} \left(E\left|X(t+h)-X(t)\right|^q\right)^{1/q} \le C_q \delta^{\tau_q};$$

where $C_p, C_q > 0$ are some constants, $\tau_p > \frac{1}{p} - \frac{1}{q}$ and $\tau_q > 0$,

then

1) for $0 < \tau_q \le 1$ the stochastic process X belongs to the Besov space $B_q^{\alpha p}[a,b]$, $0 < \alpha < \tau_q$ with probability one;

2) for $\tau_q > 1$ the stochastic process X belongs to the Besov space $B_q^{\alpha p}[a,b]$ with probability one for all $.0 < \alpha < 1$.

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φ-SUB-GAUSSIAN RISK PROCESSES

Olga Vasylyk and Rostyslav Yamnenko
Kyiv National Taras Shevchenko University
vasylyk@univ.kiev.ua, yamnenko@univ.kiev.ua

We consider a Sparre Andersen risk model

$$X(t) = u + ct - \sum_{k=1}^{N_t} Y_k ,$$

where Y_k are φ-sub-Gaussian random variables. Recall that centered random variable ξ belongs to the space of φ-sub-Gaussian random variables if for all $\lambda \in R$ there exists a constant $r_\xi \geq 0$, which satisfies the following inequality

$$E \exp\{\lambda\xi\} \leq \exp\{\varphi(\lambda r_\xi)\}.$$

The ruin probability is obtained for such a risk process.

STRONG INVARIANCE PRINCIPLE FOR RENEWAL-REWARD AND RANDOMLY STOPPED PROCESSES

Nadiia Zinchenko
Kyiv National Taras Shevchenko University, Ukraine
e-mail: znm@univ.kiev.ua

We present a number of results due to strong invariance principle for renewal(counting) processes and randomly stopped sums.

Let X, $\{X_i, i \geq 1\}$ be independent identically distributed random variables (i.i.d.r.v) with common distribution function (d.f.)$F(x)$, $EX = a$ if $E|X| < \infty$. Denote by

$$S(n) = \sum_{i=1}^{n} X_i, S(0) = 0, \quad N(t) = \inf\{n > 0 : S(n) > t\}, \tag{1}$$

$G_{\alpha,\beta}$ – stable law with parameters $0 < \alpha < 2$, $|\beta| \leq 1$ and characteristic function (ch.f.) $g_{\alpha,\beta}(u) = \exp(-K(u))$, where

$$K(u) = -|u|\big(1 - i\beta(u/|u|)\varpi(u, \alpha)\big), \tag{2}$$

where $\varpi(u, \alpha) = \tan(\pi\alpha/2)$ if $1 < \alpha < 2$, $\varpi(u, \alpha) = -(2/\pi) \log |u|$ if $\alpha = 1$, $Y(t) = Y_\alpha = Y_{\alpha,\beta}(t)$, $t \geq 0$ – **stable Lévy process** with ch.f. $g_\alpha(t; u) = \exp(tK(u))$.

Suppose that in the case $EX^2 = \infty$ following *assumption (C)* is true: there are $a_1 > 0, a_2 > 0$ and $l > \alpha$ such that for $|u| < a_1$

$$|f(u) - g_{\alpha,\beta}(u)| < a_2|u|^l$$

where $f(u)$ is a ch. f. of $(X - EX)$ if $1 < \alpha < 2$ and ch.f of X if $0 < \alpha \leq 1$.

Assumption (C) provides that $\{X_i|\}$ are in domain of normal attraction of the stable law $G_{\alpha,\beta}$.

Theorem 1. *If $\{X_i\}$ satisfy (C) with $1 < \alpha < 2$, then a.s.*

$$\left|t\lambda - N(t) - \lambda^{1+1/\alpha} Y_{\alpha,\beta}(t)\right| = o(r(t)), \tag{3}$$

where $r(t) = t^{1/\alpha+\delta}$ for any $\delta > 0$.

Horvách, M. Csörgő, Steinebach (1985 – 1996) studied a.s. approximation of the type

$$|\lambda t - N(t) - \lambda W(\lambda t)| = o(r(t)) \vee O(r(t)) \tag{4}$$

and proved that conditions which provide (4) and corresponding optimal errors are the same as for $S(n)$.

Let $S(n), N(t)$ be as above, $X_i \geq 0$ a.s., $EX = 1/\lambda$,
$\{Y_i, \geq 1\}$ – i.i.d.r.v. independent of $\{X_i, \geq 1\}$, $EY_1 = m$,

$$D(n) = \sum_{i=1}^{n} Y_i, D(0) = 0$$

$$D(N(t)) = \sum_{i=1}^{N(t)} Y_i.$$

Theorem 2. Let $\{Y_i, i \geq 1\}$ satisfy (C) with $1 < \alpha < 2$, and $EX_1^2 < \infty$ then a.s.

$$\sup_{1 \leq t \leq T} \left| D(N(t)) - m\lambda t - Y_{\alpha,\beta}(t) \right| = o(t^{1/\alpha - \varrho})$$

for some $\varrho = \varrho(\alpha, l) > 0$.

Theorem 3. Let $\{Y_i, i \geq 1\}$ satisfy (C) with $1 < \alpha_1 < 2$, and $\{X_i\}$ satisfy (C) with $1 < \alpha_2 < 2$,

$$\alpha_1 \leq \alpha_2$$

then a.s.

$$\sup_{1 \leq t \leq T} \left| D(N(t)) - m\lambda t - Y_{\alpha_1,\beta}(\lambda t) \right| = o(t^{1/\alpha_1 - \varrho})$$

for some $\varrho = \varrho(\alpha_1, l) > 0$.

All these results can be applied to investigation the asymptotic behavior of the risk processes with large claims.

CONTRIBUTED LECTURES

FINANCIAL STREAMS AS MEASURES

Richard F. Bonner
Department of Mathematics and Physics,
Mälardalen University, Västerås, Sweden.
richard.bonner@mdh.se

We start naively by noting that a "financial stream" is something that tells us how much money has changed hands in any given period of time, and that this function of time-intervals is additive. Such an object is effectively a measure. That a (Borel) measure on the real line could be made to model a stream is of course obvious once it has been thought of as a generalized function (Schwartz distribution). It is also straightforward that no larger space of distributions can model streams if the (Jordan) decomposition of a stream into a positive and a negative part should remain valid.

Remains, however, the question: how natural is it to use the full structure of a measure space in support of basic financial analysis? It indeed does appear natural. We here consider this question in a non-random setting, extension to random measures being formally straightforward. The measure formalism appears useful in several ways, in particular for orthodox interpretation of financial notions and their symbolic manipulation, and for handling infinite-dimensional optimisation problems. It should also be useful in education, financial analysis then being a natural application of measure theory, and conversely, it should help financial analysts to find measure theory convincing.

LEARNING A REAL NUMBER, RATIONALLY

Richard Bonner and Anna Fedyszak-Koszela
Department of Mathematics and Physics,
Mälardalen University, Västerås, Sweden.
richard.bonner@mdh.se, anna.koszela@mdh.se

We are interested in economics of learning, especially of the generic "probably approximately correctly" (PAC) learning. In particular, we wish to rationally bound the stopping time for learning from random samples. An orthodox PAC learner appears naturally rational in the well-known maxmin sense of Gilboa and Schmeidler, removing inherent ambiguity in the prior by minimization: the model bounds from below the stopping time in terms of the largest expected error of recall, and thus, effectively, in terms of the least expected reward. We briefly discuss the grounds for rational decision in face of ambiguity of the kind characteristic to PAC "batch" models. We then look at possibly the simplest yet not entirely trivial problem of this type of learning, the problem of locating a real number θ by testing whether $x_i < \theta$, with x_i drawn from an unknown probability measure. Bounds for the stopping time are here immediate, and we display these in function of term rate, sample cost, and reward/penalty from a recall. We end by pointing to standard econometric situations, such as product promotion, market research, credit risk assessment, and bargaining and tenders, where such bounds could be of interest.

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EXPLICIT SOLUTION OF THE FUNDAMENTAL EQUATION OF RISK THEORY FOR DISCRETE MODEL

Chernecky V.A.
Department of Mathematical Analysis
Odessa I.I.Mecnikov National University
chern.va@paco.net

Let $T_1, T_2, ...,$ be a sequence of mutually independent random variables, the interarrival times, with a common generating function $g_T(z) = \sum_{k=1}^{\infty} q_k z^k$ and mean value $ET_k = 1/\alpha$, $Z_1, Z_2, ...,$ be a sequence of mutually independent random variables, the claims to an insurance company, with a common generating function $g_Z(z) = \sum_{k=1}^{\infty} p_k z^k$ and mean value $EZ_k = \mu$, and $c > \alpha\mu$ be the gross premium rate, $c \in \mathbf{N}$. The non-ruin probabilities φ_u of the insurance company with initial capital $u \in \mathbf{Z}$ satisfies the infinite system of linear algebraic equations of Wiener-Hopf type,

$$\varphi_u = \sum_{v=1}^{\infty} q_v \sum_{z=1}^{u+cv} \varphi_{u+cv-z} p_z \,, \quad \lim_{u \to +\infty} \varphi_u = 1 \,, \tag{1}$$

that is the discrete analog of the Lundberg-Feller equation. Note that the unitial capital u of the company can accept as non-negative as negative integers. The non-ruin probability φ_u in the latter case means the probability that the company begins the activity having a debt $(-u)$, however to the moment of the first claim the capital of the company becomes positive and is hereinafter not below of zero.

For the solution of the equation (1) the discrete analogue of the Wiener-Hopf method is applied. The symbol of the equation (1) is calculated by the formula

$$A(t) = 1 - g_T(t^{-c}) \cdot g_Z(t), \qquad |t| = 1.$$

The symbol $A(t)$ certainly has zero of the first order at the point $t = 1$ and can have or not have the finite number of zero on the unite circle $L = \{t : |t| = 1\}$ in dependence on particular problem. The solution of the equation (1) is reduced to the solution of the Riemann boundary value problem on the unite circle L

$$\varphi^+(t) = \frac{1}{A(t)} \varphi^-(t), \qquad t \in L, \tag{2}$$

so the required probabilities φ_u, $u \in \mathbf{Z}$, in (1), are the coefficients of the (formal) Laurent series of the function $\varphi^+(t) - \varphi^-(t)$. The coefficient $\frac{1}{A(t)}$ of the Riemann boundary value problem (2) has the pole of the first order on the contour L in the point $t = 1$ and we search for all solutions generated only by this singularity. The analysis shows that the problem (2) has two linearly independent solutions: $\varphi_1^{\pm}(t)$ for which $\varphi_1^+(t)$ has the pole of the first order at the point $t = 1$, and a solution $\varphi_2^{\pm}(t)$ not possessing singularity in the point $t = 1$. The (real) coefficients of the Taylor series of the function $\varphi_1^+(z)$ at the point $z = 0$, monotonically growing aim for a finite limit, and the coefficients of the function $\varphi_2^+(z)$ monotonically decreasing aim for zero.

The general solution of the problem (2) is of the form

$$\varphi^{\pm}(t) = C_1 \cdot \varphi_1^{\pm}(t) + C_2 \cdot \varphi_2^{\pm}(t).$$

(3)

The accompanying stationary renewal epochs $S_n = S_0 + T_1 + ... + T_n$ with the generating function

$$g_{S_0}(z) = \frac{\alpha \cdot z \cdot (1 - g_T(z))}{1 - z}$$

for S_0 is introduced, and the generating function $\varphi^s(z)$ for non-ruin probabilities in the stationary case is built as

$$\varphi^s(z) = g_{S_0}(z^{-c}) \cdot g_Z(z) \cdot \varphi^+(z),$$

similarly as it is done in the non-arithmetic case. So the unknown coefficients C_1 and C_2 in (3) are found from two conditions

$$\lim_{u \to +\infty} \varphi_u^+ = 1 \quad \text{and} \quad \lim_{u \to +\infty} \varphi_u^s = 1,$$

where φ_u^s are the coefficients of the Laurent series of the function $\varphi^s(t)$.

Number of examples when T_k and Z_k have shifted geometrical or shifted negative binomial (with natural parameter) distributions is considered.

DELAYED RENEWAL PROCESSES AND STATIONARITY IN ARITHMETIC CASE

Chernecky V.A.
Department of Mathematical Analysis
Odessa I.I.Mecnikov National University
chern.va@paco.net
Pokas' S.M.
Department of Geometry and Topology
Odessa I.I.Mecnikov National University
math@imem.odessa.ua

Let $T_1, T_2, ...,$ be a sequence of mutually independent lattice random variables, the interarrival times, with a common generating function $g_T(z) = \sum_{n=1}^{\infty} p_n z^n$ and mean value $ET_n = 1/\alpha < \infty$. In addition to the T_k there is defined a lattice variable S_0 with a generating function $g_0(z) = \sum_{n=1}^{\infty} q_n z^n$. Consider renewal epochs $S_n = S_0 + T_1 + ... + T_n$. The renewal process $\{S_n\}$ is called delayed if $S_0 \neq 0$. The expected number $V(t) = \sum_{n=0}^{\infty} P\{S_n \leq t\}$ of the renewal epochs in $[0, t]$ has the generating function

$$g_V(z) = \frac{g_0(z)}{(1-z)(1-g_T(z))}.$$

If T_k are arithmetic, the expected number of renewal epochs within $[t, t+n]$, $n \in \mathbf{N}$, tends to $\alpha \cdot n$. It follows that $V(n) \approx \alpha \cdot n$ as $n \to \infty$. It is natural to ask whether $g_0(z)$ can be chosen as to get the identity $V(n) = \alpha \cdot n$, $n \in \mathbf{N}$, meaning a constant renewal rate. Noticing that the generating function for $V(n) = \alpha \cdot n$, $n \in \mathbf{N}$, equals

$$g_V(z) = \frac{\alpha \cdot z}{(1-z)^2}$$

we have

$$g_0(z) = (1 - g_T(z)) \cdot g_V(z) \cdot (1-z) = \frac{\alpha \cdot z \cdot (1 - g_T(z))}{1-z}. \qquad (1)$$

This $g_0(z)$ is a generating function of a lattice random variable and so the answer is affirmative: with the initial random variable S_0 having the generating function (1) the renewal rate is constant,

$$V(n) = \alpha \cdot n, \ n \in \mathbf{N}.$$

The following statement takes place:

The stationary renewal process in arithmetic case is ordinary if and only if, when the interarrival times T_k have the shifted geometrical distribution with the generating function

$$g_T(z) = \sum_{n=1}^{\infty} p(1-p)^{n-1} z^n.$$

This shifted geometrical distribution in arithmetic case is the analog of exponential distribution in non-arithmetic case.

ГРАНИЧНІ ТЕОРЕМИ ДЛЯ ВАГОВИХ ФУНКЦІОНАЛІВ ВІД ВИПАДКОВИХ ПОЛІВ

Б. Кликавка
КНУ ім. Тараса Шевченка
Україна

В теорії випадкових функцій важливою задачею є отримання граничних теорем для функціоналів від випадкових полів. Теореми становлять не тільки самостійний інтерес але й відіграють суттєву роль в задачах статистики. Важливу роль при отриманні багатьох результатів такого типу відіграють абелеві і тауберові теореми.

Нехай $\xi(t),\ t \in R^n$ - дійсне вимірне неперервне в середньому квадратичному однорідне та ізотропне в широкому розумінні випадкове поле з нульовим математичним сподіванням і кореляційною функцією $B_n(r)$. Нехай $\Phi(\lambda),\ \ \lambda \geq 0$ - спектральна функція поля $\xi(t),\ t \in R^n$. Розглянемо функцію $\Phi_a(\lambda),\ \ \lambda \geq 0$ визначену таким чином

$$\Phi_a(\lambda) := \Phi(a+\lambda) - \Phi(a), \quad \lambda \geq 0, a \geq 0.$$

У роботі [1] було показано, що існує дійснозначна функція $g_{n,r,a}(\cdot)$ така, що

$$\tilde{b}_a(r) := (2\pi)^n \int_0^\infty \frac{J_{n/2}^2(rx)}{(rx)^n} d\Phi_a(x) = D\left[\int_{\mathbb{R}^n} g_{n,r,a}(|t|)\xi(t)dt\right].$$

Досліджено властивості вагового функціоналу: одержано швидкість збіжності функціональних рядів у зображенні функції $g_{n,r,a}(t)$, знайдено рекурентну формулу зв'язку вагових функцій у просторах різної розмірності.

Вивчено асимптотичні властивості вагових функцій, зокрема при $n \geq 1$

$$\lim_{r\to\infty} g_{n,r,a}(|t|) = 0, \qquad \lim_{|t|\to\infty} g_{n,r,a}(|t|) = 0.$$

На основі отриманих результатів одержано граничні теореми для інтегралів вагових функціоналів від випадкових полів

$$\int_{\mathbb{R}^n} g_{n,r,a}(|t|)\xi(t)dt.$$

Ці результати узагальнюють відомі результати для функціоналів середніх по кулі та сфері.

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РОЗРОБКА СТРУКТУРНИХ МОДЕЛЕЙ УКРАЇНСЬКОЇ ЕКОНОМІКИ

В.Л. Плескач
Науково-дослідний фінансовий інститут Міністерства фінансів України
e-mail: Pleskach_Valentina_L@ndfi.kiev.ua
Р.Є.Майборода
Київський національний університет імені Тараса Шевченка
e-mail: mre@univ.kiev.ua

Доповідь присвячена підходам до моделювання української економіки, які розробляються у Науково-дослідному фінансовому інституті Міністерства фінансів України. Розглядаються загальні методики моделювання макроекономічних процесів та створення цілісних моделей національної економіки. Головна увага приділяється структурним моделям.

На математичному рівні структурні моделі являють собою систему одночасних рівнянь, що описують економіку країни, або деяку її частину. Ці рівняння бувають двох типів – стохастичні рівняння (або рівняння поведінки) та рівності. Стохастичні рівняння описують окремі закономірності економічної динаміки, як правило, на основі певних теоретичних припущень. Коефіцієнти (параметри) цих рівнянь оцінюються на основі історичних даних методами математичної статистики. Рівності – це рівняння, які виконуються за означенням. Як правило, рівності є або рівняннями балансу певних грошових потоків, або означеннями розглядуваних економетричних змінних.

Обсяг наявних даних про функціонування економіки реальних країн помітно обмежує можливості оцінювання багатьох параметрів одразу у системах одночасних рівнянь. З іншого боку, моделі, що спираються на порівняно невелику кількість рівнянь, не можуть описувати складні явища реальної економіки з достатньою точністю. Стандартним підходом до розв'язання цієї проблеми є розбиття загальної моделі економіки на структурні блоки, зв'язки між якими є менш сильними у порівнянні із зв'язками всередині блоку. Це дозволяє всередині блоку наближено розглядати змінні з інших блоків як екзогенні і оцінювати параметри окремо для систем рівнянь кожного блоку. Зрозуміло, що при розбиття системи на блоки можна проводити на різних засадах, але в будь-якому випадку воно повинно бути обґрунтованим економічними міркуваннями. Класичним є виділення таких блоків економіки, як сектори домогосподарств, підприємств, фінансовий, державний і закордонний.

У доповіді обговорюються можливості застосування цього підходу до моделювання української економіки та результати аналізу статистичних даних.

ДЕЯКІ СТАТИСТИЧНІ АСПЕКТИ СТАТИСТИЧНОЇ ОСВІТИ

Мамчич Т.І.
Волинський державний університет імені Лесі Українки
(м. Луцьк)

Статистичні методи, як методологія емпіричних досліджень, присутні в навчальних планах підготовки багатьох спеціальностей університетів. Ця дисципліна має певні традиції щодо змісту курсу та організації навчання. Але в умовах комп'ютеризації наукових досліджень та навчального процесу виникає потреба модернізації всіх аспектів системи навчання.

Зрозуміло, що різні спеціальності мають відмінні потреби у використанні різних статистичних методів. Для з'ясування специфіки фаху щодо змісту навчального курсу із прикладної статистики було використано ресурси мережі Інтернет. Вивчались частоти звернення до тих чи інших статистичних методів фахівців з різних галузей.

Також проаналізовано досвід діяльності лабораторії проблем прикладної статистики Волинського держуніверситету. Відзначено підвищення рівня загальної компетентності фахівців з питань, пов'язаних з обробкою даних та зміну характеру звернень за консультаціями.

Пропонується результати дослідження використовувати при проектуванні комп'ютерних програм з контролю знань та навчаючих програм.

MODELLING OF RANDOM PROCESS WITH KNOWN CORRELATION FUNCTION WITH THE HELP OF KARHUNEN-LOEVE DECOMPOSITION

O. M. Moklyachuk
Kiyv Taras Shevchenko National University
e-mail:outlander@a.ua

Gaussian processes are good in describing many problems in financial and actuarial mathematics. Modelling of Gaussian processes has many applications in this field.

Let $\{X(t), t \in [0,T]\}$ be a centred Gaussian stochastic process with the known correlation function $B(t,s) = B(t-s) = EX(t)X(s)$. Consider the Karhunen-Loeve decomposition of the process:

$$X(t) = \sum_{k=1}^{\infty} \xi_k \frac{\varphi_k(t)}{\sqrt{\lambda_k}},$$

where $\xi_k \sim N(0,1)$, $\varphi_k(t)$ are eigenfunctions of the equation $\varphi(t) = \lambda \int_0^T B(t,s)\varphi(s)ds$, and λ_k are the corresponding eigenvalues. Let $\hat{\varphi}_k(t)$ be approximations of eigenfunctions and let $\hat{\lambda}_k$ be approximations of eigenvalues such that $|\hat{\lambda}_k - \lambda_k| < \delta_k$ and $\|\hat{\varphi}_k(t) - \varphi_k(t)\| < \alpha_k$. Determine the model $X_N(t)$ of the process $\{X(t), t \in [0,T]\}$ using these approximations:

$$X_N(t) = \sum_{k=1}^{N} \xi_k \frac{\hat{\varphi}_k(t)}{\sqrt{\hat{\lambda}_k}},$$

To find approximations of eigenfunctions and eigenvalues we use the method of approximate integration. We take n points $x_i = \frac{iT}{n}$, i=1,…,n from the segment (0, T) and build the matrix A, where $a_{ij} = B(x_i, x_j)$. We search for the first n eigenvalues $\hat{\lambda}_k$ of the matrix A. Next we find approximations of eigenfunctions $\hat{\varphi}_k(t)$ using the method of rectangles.
We propose formulas for the estimation of the error of approximation $X(t)$ by the model $X_N(t)$ in the space $L_p(0,T)$.

Algorithms of modelling and estimation are created using Mathematica 6.0 software.

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TIME SERIES MODEL SELECTION OF THE FARIMA CLASS IN FINANCIAL MATHEMATICS

Mikhail Moklyachuk, Aleksey Zrazhevsky
Kyiv National Taras Shevchenko University
mmp@univ.kiev.ua, zalex@univ.kiev.ua.

The problem of estimation of the dimension of a model occurs in various forms in applied statistics. There are estimation of the degree of a polynomial describing data, selection variables to be introduced in a multiple regression equation, estimation the order of an AR or MA time series models.

Many financial time series demonstrate long-range dependence with/without the presence of short-range dependent components ([1]-[3]). Problems of simulation and prediction of such time series reduce to necessity of extension of the model selection criterions on the case of long-memory data. This presentation deals with selection of an appropriate model for financial time series from the class of FARIMA models. The considered problem may be reduced to the problem of estimation of parameters of the FARIMA(p,d,q) model with differencing exponent d and orders of the AR and the MA polynomials p and q correspondingly. Some methods of estimation of the exponent parameter d are obtained using the Hurst parameter. Another problem is connected with the estimation of parameters p and q. These parameters can be estimated using the Akaike information criterion (AIC, AICC), the Bayesian information criterion (BIC), the Hannan-Quinn criterion (HIC) in the case of the ARMA model (d=1) ([3]-[4]). For the FARIMA model ($d \neq 1$) it is impossible to identify the order of the short memory polynomials p and q by using the classical methods. We demonstrate the method of the aggregation of data with the autoregressive filter to extract the long-range dependent components. This technique supposes that the investigated time series is a realization of the FARIMA model and does not produce the appropriate results otherwise. These results are justified by numerical researches on the basis of simulated FARIMA time series with the different parameters and the observed financial time series (MSFT ticker data).

The problem of the model selection for the long-memory time series in general case can be solved through extrapolation of the classical one or some new methods without using the assumption about the exact distribution of the underlying process and the hypothesis about presence of AR or/and MA components should be applied only.

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SOME CLASSES OF INVARIANT SECOND-ORDER RANDOM MEASURES

Oleksandr Ponomarenko
Kyiv Taras Shevchenko National University
e-mail: probability@univ.kiev.ua
Yuriy Perun
National Bank of Ukraine
e-mail: perun@bank.gov.ua

In modern spectral theory of second-order random functions (processes and fields) and its statistical applications we can meet the examples of random measures with some invariant properties. This communication is devoted to systematical studying of some classes of invariant second-order random measures on σ-algebra of Borel sets in n-dimensional space $R^n, n \ge 1$. The invariance of such measures is defined in the terns of associated covariance complex valued bimeasures. We consider the class of translation invariant measures, the class of random measures which are invariant to respect of all hard movements of R^n (i.e. translations and rotations) and the class of random measures with exponentially convex property. The structure representations theorems for such random measures which characterize their relations with corresponding classes of invariant random functions are obtained. Some generalizations of these results to case of multivariate random measures on Abelian locally compact groups are also considered.

ASYMPTOTIC EXPANSIONS FOR STATIONARY DISTRIBUTIONS OF NONLINEARLY PERTURBED MARKOV CHAINS

Dmitrii Silvestrov
Mälardalen University
Västerås, Sweden
E-mail: dmitrii.silvestrov@mdh.se
Zakir Abadov
zakirabadov@box.az

Asymptotic expansions are given for stationary distributions of nonlinearly perturbed exponentially ergodic Markov chains. The expansions are provided by explicit estimates for reminder terms.